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AN ANALYSIS OF INTERNAL FORCES IN A SIMPLE TRUSS STRUCTURE: GAUSS-JORDAN ELIMINATION METHOD AND THE EFFECT OF NUMERICAL ERRORS

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ABSTRACT

This study analyzes the internal forces in a simple truss structure using the Gauss-Jordan method. The truss structure is analyzed through a mathematical approach by formulating a mathematical model of the simple truss structure in the form of a system of linear equations based on the equilibrium of forces at each joint. The resulting system of force equations is transformed into a coefficient matrix and solved using the Gauss-Jordan method to determine the internal forces in each truss member. This study also evaluates the impact of numerical errors in computation. The results indicate that the Gauss-Jordan method is effective for small-scale structures, but it is sensitive to rounding errors and near-zero coefficients, thus highlighting the need for careful application of numerical methods in civil structural analysis.

Keywords: Gauss-Jordan Method, Internal Force Analysis, Numerical Error, Truss Structure

Introduction

A truss is a type of structural element composed of thin profiles connected at both ends. Channels, angles, metal bars, and wooden *struts* are materials commonly used in building construction. At the end of the elements, they are usually bolted or welded to plates called *gusset plates*[1] .

A truss structure, also known as a jointed structure, is a structure consisting of straight rods connected using friction joints[2]. Structural analysis is the study of how a structure responds to external forces such as loads, temperature variations, and supports[3].

Internal force analysis in simple truss structures is an important step to ensure the stability and reliability of structures in civil engineering and structural mechanics. Through internal force analysis, truss designs can be made more efficiently and economically. Research[4] shows that truss analysis using the nodal equilibrium method and the Ritter method does not show any differences in calculation results. By ensuring that each structural element only receives forces that are within its capacity, excessive use of materials can be avoided, which not only reduces costs but also minimizes the waste of natural resources and supports the principle of sustainability in construction.

One of the most commonly used methods for internal force analysis is the Gauss-Jordan method[5], which is a numerical technique for solving systems of linear equations that describe the equilibrium of forces in a structure. Gauss-Jordan elimination is also used in solving GPS problems[6], balancing chemical reactions using systems of linear equations[7], and determining currents in electrical circuits[8].

Although this method is known for its efficiency, there are challenges that are often encountered, namely numerical errors that can arise during the calculation process[9]. These errors can affect the accuracy of the analysis results, especially when dealing with complex structures or when the number of elements in the truss increases.

The purpose of this study is to evaluate the effectiveness of the Gauss-Jordan method in analyzing internal forces in simple truss structures, as well as to analyze how numerical errors can affect the results of internal force calculations. This study is expected to contribute to the development of more efficient and accurate methods in structural engineering.

Method

This study was conducted by compiling and applying a basic mathematical model to calculate internal forces on simple triangular truss members using the Gauss-Jordan method. The simple truss structure to be analyzed is shown in Fig. 1. below:

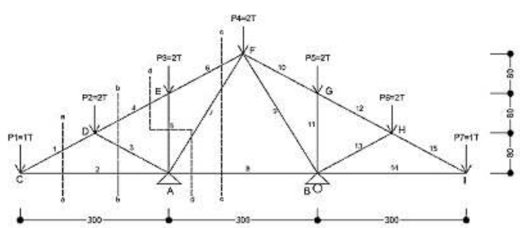


Fig. 1. Simple truss structure

The analysis steps carried out in this study are:

1. Developing a mathematical model for a simple truss structure as seen follows:
    a. Identifying the elements of the simple truss structure
    b. Formulating force equilibrium equations for each joint
    c. Developing a system of linear equations
    d. Converting the system of equations into matrix
2. Implementing the Gauss Jordan method to analyze simple truss structures as seen below:
    a. Forming the coefficient matrix from the system of force equations that have been formed
    b. Performing Gauss Jordan elimination to reduce the coefficient matrix
    c. Solving the system of equations to obtain the internal forces in each truss
3. Analyzing the numerical error

Results

This study used a frame structure which has 15 stem and 9 joints with support reaction forces was used, as seen below:

$$RA_v = RB_v = \frac{1}{2}(1 + 2 + 2 + 2 + 2 + 2 + 1) = 6\ ton \quad (1)$$

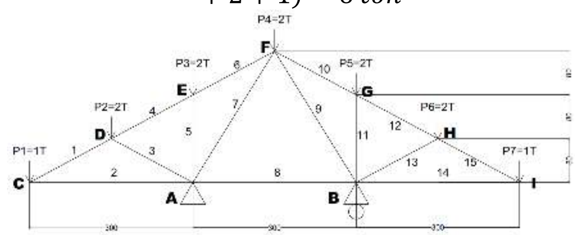


Fig. 2. Simple truss structure

The identification of forces on each stem is shown in the following figure:

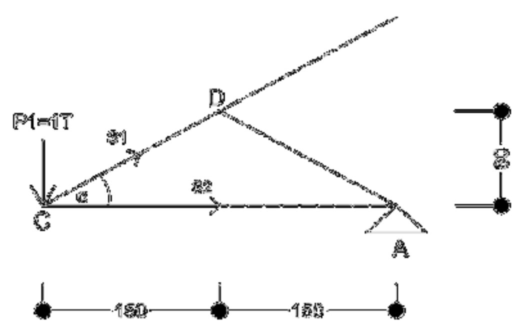
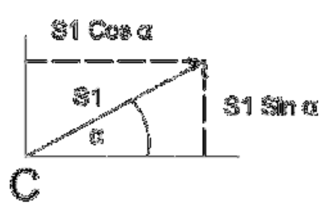


Fig. 3. Section a-a



$$\tan\alpha = \frac{80}{150} = 0{,}5333$$

$$\alpha = arc\tan 0{,}5333 = 28°$$

$\sin 28° = 0{,}470$

$\cos 28° = 0{,}833$

$\sum M_D = 0$

$$-P_1.150 - S_2.80 = 0 \quad (2)$$

$S_2 = -\frac{150}{80} = -1{,}875\ Ton$ (**Press**)

$\sum M_A = 0$

$$-P_1.300 + S_1.\sin\alpha.300 = 0 \quad (3)$$

$(-1).300 + S_1.0{,}470.300 = 0$

$S_1 = \frac{300}{141} = 2{,}128\ Ton$ (**Pull**)

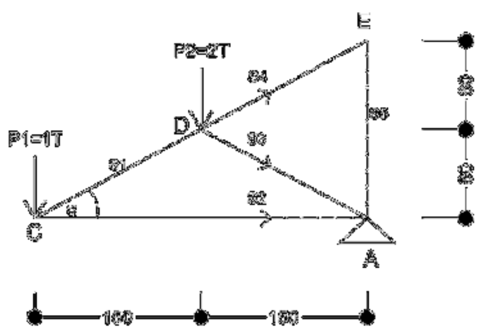
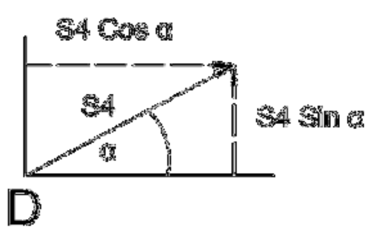


Fig. 4. Section b-b

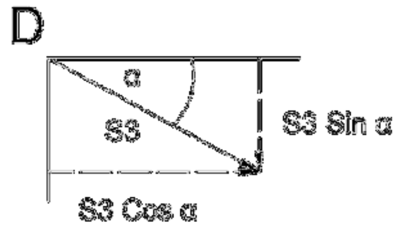


$$-P_1.300 + S_4.\sin\alpha.150 + S_4.\cos\alpha.80 - P_2.150 = 0 \quad (4)$$

$$(-1).300 + S_4.0{,}470.150 + S_4.0{,}883.80 - 2.150 = 0$$

$$S_4 = \frac{600}{70{,}5 + 70{,}64} = \frac{600}{141{,}14}$$

$S_4 = 4{,}251\ Ton$ (**Pull**)



$\sum M_c = 0$

$$P_2.150 + S_2.\sin\alpha.150 + S_3.\cos\alpha.80 = 0 \quad (5)$$

$(2).150 + S_3.0{,}470.150 + S_3.0{,}883.80 = 0$

$S_3 = -\frac{300}{141{,}14} = -2{,}128\ Ton$ (**Press**)

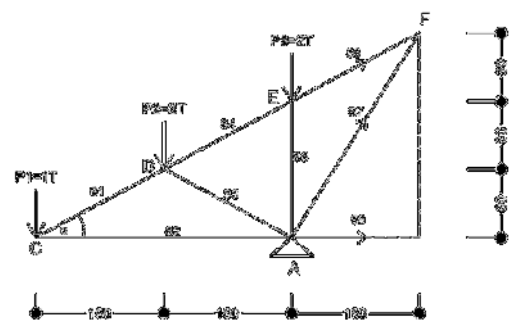


Fig. 5. Section c-c

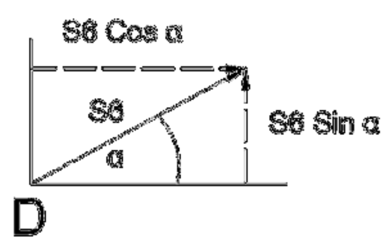
$\sum M_F = 0$

$$-P_1.450 - P_2.300 - P_3.150 + R_{AV}.150 - S_8.240 = 0 \quad (6)$$

$$(-1).450 - 2.300 - 2.150 + 6.150 - S_8.240 = 0$$

$-450 - 600 - 300 + 900 - S_8.240 = 0$

$S_8 = -\frac{450}{240} = -1{,}875\ Ton$ (**Press**)

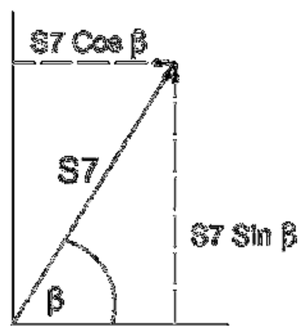


$$\sum M_A = 0$$

$$-P_1.300 - P_2.150 + S_6.\cos\alpha.160 = 0 \quad (7)$$

$$(-1).300 - 2.150 + S_6.0{,}883.160 = 0$$

$$S_6 = \frac{600}{141{,}28} = 4{,}274\ Ton\ (\textbf{Pull})$$



$$\tan\beta = \frac{240}{150} = 1{,}6$$

$$\beta = arc\tan 1{,}6 = 57{,}995°$$

$$\sin\beta = \sin 57{,}995° = 0{,}848$$

$$\cos\beta = \cos 57{,}995° = 0{,}530$$

$$\sum M_E = 0$$

$$-P_1.300 - P_2.150 - S_8.160 - S_7.\cos\beta.160 = 0 \quad (8)$$

$$(-1).300 - 2.150 - (-1{,}875).160 - S_7.0{,}530.160 = 0$$

$$-300 - 300 + 300 - S_7.84{,}8 = 0$$

$$S_7 = -\frac{300}{84{,}8} = -3{,}538\ Ton\ (\textbf{Press})$$

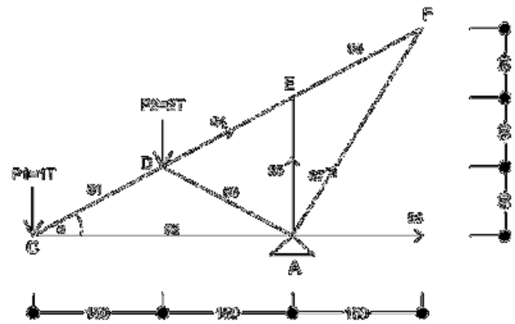


Fig. 6. D-d section

$$\sum M_F = 0$$

$$-\mathrm{P}_1.450 - \mathrm{P}_2.300 + \mathrm{R}_{AV}.150 - \mathrm{S}_8.240 + \mathrm{S}_5.150 = 0 \quad (9)$$

$$(-1).450 - 2.300 + 6.150 - (-1{,}875).240 + S_5.150 = 0$$

$$-450 - 600 + 900 + 450 + S_5.150 = 0$$

$$S_5 = -\frac{300}{150} = -2\ Ton\ (\textbf{Press})$$

Because the frame is symmetrical, calculations only need to be performed on beams 1 to 8. Table 1 shows the results of calculations using the Ritter method.

Table 1. Calculation results using the ritter method

Member Number	Member Force (Tension)	Member Force (Compression)
1 = 15	2,128	-
2 = 14	-	1,875
3 = 13	-	2,128

4 = 12	4,251	-
5 = 11	-	2,000
6 = 10	4,247	-
7 = 9	-	3,538
8	-	1,875

Discussion

In solving using the Gauss-Jordan elimination method, the coefficient matrix of the force equations formed in equations (1) to (8) is arranged into the approriate matrix form as follows.

$$\begin{bmatrix} 0 & -80 & 0 & 0 & 0 & 0 & 0 & 0 \\ 140{,}841 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 141{,}057 & 0 & 0 & 0 & 0 \\ 0 & 0 & 141{,}057 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -240 \\ 0 & 0 & 0 & 0 & 0 & 141{,}672 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -84{,}799 & -160 \\ 0 & 0 & 0 & 0 & 150 & 0 & 0 & -240 \end{bmatrix} \cdot \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{bmatrix} = \begin{bmatrix} [illegible] \\ [illegible] \\ [illegible] \\ [illegible] \\ [illegible] \\ [illegible] \\ [illegible] \\ [illegible] \end{bmatrix}$$

The next step is to perform Gauss-Jordan elimination to reduce the coefficient matrix to an identity matrix with the following steps:

1. $\frac{1}{-80}R_1 \to R_1$
2. $\frac{1}{140{,}841}R_2 \to R_2$
3. $\frac{1}{141{,}057}R_3 \to R_3$
4. $\frac{1}{141{,}057}R_4 \to R_4$
5. $\frac{1}{-240}R_5 \to R_5$
6. $\frac{1}{141{,}672}R_6 \to R_6$
7. $160R_5 + R_7 \to R_7$
8. $\frac{1}{-84{,}799}R_7 \to R_7$
9. $(-1)R_5 + R_8 \to R_8$
10. $\frac{1}{150}R_8 \to R_8$

The result of Gauss-Jordan elimination is an identity matrix as follows:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{bmatrix} = \begin{bmatrix} -1{,}875 \\ 2{,}1300544682 \\ 4{,}2536134244 \\ -2{,}1268067122 \\ -1{,}875 \\ 4{,}2471376901 \\ -3{,}5377807831 \\ -2 \end{bmatrix}$$

From the above calculations, the internal forces are obtained as follows:

$S_1 = 2{,}1300544682$
$S_2 = -1{,}875$
$S_3 = -2{,}1268067122$
$S_4 = 4{,}2536134244$
$S_5 = -2$
$S_6 = 4{,}2471376901$
$S_7 = -3{,}5377807831$
$S_8 = -1{,}875$

Numerical errors can arise due to inherent errors, rounding errors, and truncation errors. Relative errors are written as a percentage comparing the absolute error and the actual value[10] . In this study, relative errors were calculated by comparing the difference in values between manual calculations and calculations using the Gauss-Jordan elimination method for various roundings (14 decimal places, 7 decimal places, 5 decimal places, and 3 decimal places) as shown in the following figure:

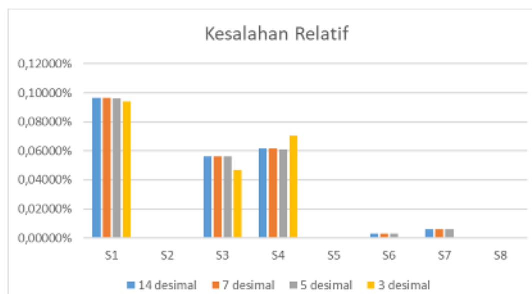


Fig. 7. Relative error levels

Based on the results of the relative error level calculations, it can be seen that there are no significant differences for rounding to 14 decimal places, 7 decimal places, and 5 decimal places. Meanwhile, in the 3 decimal rounding, the relative error rate decreased for S_1, S_3, S_6, and S_7 values. For S_2, S_5, and S_8 values, there was no change because there was no difference between the manual calculation results and the Gauss-Jordan method calculation results. For

S_4 value, there was an increase in the relative error rate between the 5 decimal and 3 decimal rounding, which was 0.00916%.

Conclusion

Based on the results of the internal force analysis on a simple truss structure using the Ritter method, there is a slight difference between the manual calculation results and the calculation using the Gauss-Jordan elimination method. The difference arises because of numerical errors. One of which is due to the rounding process. The largest relative error difference is seen in the calculation results with 3 decimal places. Meanwhile, with 14 decimal places, 7 decimal places, and 5 decimal places, the differences are very small.

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